PROPAGATION OF WEAK PERTURBATIONS IN TWO-PHASE MEDIA WITH PHASE TRANSITIONS

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The linearized equations of fluid mechanics [1] and the equation of state of the phases are used to investigate the propagation of weak perturbations in two-phase media which are a mixture of a gas and drops or particles. Allowance is made for possible phase transitions. The dependences of the wave vector on the perturbation frequency are obtained. An estimate is made of the effect of mass exchange between the phases on the nature of the dispersion relations. Some theoretical and experimental investigations devoted to the propagation of sound in two-phase media have been made, for example, in [2-5]. Throughout the paper the quantities that refer to the gas and the particles carry the subscripts 1 and 2, respectively. The subscript 0 refers to the unperturbed state; the subscript 3 to the saturation state.

1. The equations of conservation of mass, momentum, and energy of the phases [1] take the following form after linearization:

$$\frac{\partial\rho_1}{\partial t}+\rho_{10}\operatorname{div}\mathbf{v}_1=J^\circ-J_0,\quad \frac{\partial\rho_2}{\partial t}+\rho_{20}\operatorname{div}\mathbf{v}_2=J_0-J^\circ$$
$$\rho_{10}\frac{\partial\mathbf{v}_1}{\partial t}=-\alpha\nabla p-\rho_{20}\mathbf{f},\quad \rho_{20}\frac{\partial\mathbf{v}_2}{\partial t}=-(1-\alpha)\nabla p+\rho_{20}\mathbf{f} \tag{1.1}$$
$$\rho_{10}\frac{\partial i_1}{\partial t}+\rho_{20}\frac{\partial i_2}{\partial t}=\frac{\partial p}{\partial t}+(J_0-J^\circ)\,l_0,\quad \rho_{20}\frac{\partial e_2}{\partial t}=\rho_{20}q-J^\circ l_0+J_0p_0\left(\frac{1}{\rho_{10}^\circ}-\frac{1}{\rho_2^\circ}\right)$$

Here ρ, p, i, and e are the perturbations of the mean density, pressure, enthalpy, and internal energy; α is the volume concentration of the gas; l is the specific heat of vaporization; and J_0 and J° are, respectively, the observed rates of condensation and vaporization in unit volume of the mixture.

If the relations are given that reflect the force interaction f, the heat exchange q, and the mass exchange J_0 and J°, and if the equations of state of the phases are also given, the system (1.1) is closed. For f and q, we can take the following linear relations, which are valid for laminar flow around an isolated sphere (for sufficiently small numbers N_{Re} of the relative flow):

$$\mathbf{f}=\frac{18\mu_1}{\rho_2^\circ d^2}(\mathbf{v}_1-\mathbf{v}_2),\qquad q=\frac{12k_1}{\rho_2^\circ d^2}(T_1-T_2) \tag{1.2}$$

Here μ_1 is the viscosity; k_1 is the coefficient of heat transfer of the material of the first phase; ρ_2° is the density of the material of the drops (particles) and d is their diameter.

The effect of the volume of the particles on the frictional force between the phases in the region of small N_{Re} can be taken into account by means of an additional factor [6, 7]:

$$\varphi=(1/\alpha)^{3.75} \tag{1.3}$$

The equations of the kinetics of the phase transitions when there are small superheatings or supercoolings $(T_i-T_3)/T_3$ can be written in the form [1]

$$J_0=\frac{6(1-\alpha)\,l}{T_0d}F_0(T_3-T_1),\quad J^\circ=\frac{6(1-\alpha)\,l}{T_0d}F^\circ(T_2-T_3) \tag{1.4}$$

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where F_0 and F° are coefficients determined experimentally or by other considerations.

Note that these equations can be used if the linear dimension of the perturbations (the wavelength) is much greater than the drop diameter and the perturbation amplitudes are sufficiently small.

2. For a cone-component two-phase medium we introduce the following equations of state of the phases:

$$p = \rho_1^\circ R_1 T_1,\quad i_1(p, T) = c_2(T_3 - T^\circ) + p/\rho_2^\circ + l(p) + c_{p1}(T_1 - T_3)$$
$$\rho_2^\circ = \text{const},\quad i_2(p, T) = c_2(T_2 - T^\circ) + p/\rho_2^\circ \quad (c_{p1},\ c_2 = \text{const}) \tag{2.1}$$

We introduce the dimensionless variables

$$P = \frac{p}{p_0},\quad U = \frac{v}{a},\quad \theta = \frac{T}{T_0},\quad \Phi = \frac{\rho}{\rho_2^\circ} \quad \left(a^2 = \frac{\gamma p_0}{\rho_{10}^\circ}\right) \tag{2.2}$$

and the parameters

$$C_1 = \frac{c_{p1}}{\gamma R_1},\quad C_2 = \frac{c_2}{\gamma R_1},\quad \theta_3 = \frac{T_3}{T_0},\quad L = \frac{l_0}{a^2},\quad r = \frac{\rho_{10}^\circ}{\rho_2^\circ} \tag{2.3}$$

and also the reduced variables

$$f^* = \frac{\rho_{20} f}{\rho_2^\circ a},\quad q^* = \frac{\rho_{20} q}{\rho_2^\circ a^3},\quad J^* = \frac{J^\circ}{\rho_2^\circ a},\quad J_* = \frac{J_0}{\rho_2^\circ a},\quad \tau = at \tag{2.4}$$

Here a is the velocity of sound in the first phase and τ is the reduced time in units of length.

Linearizing (2.1) and using (1.1)-(1.4) and (2.2)-(2.4), we obtain the following system of equations (one-dimensional case):

$$\frac{\partial \Phi_1}{\partial \tau} + \alpha r \frac{\partial U_1}{\partial x} = J^* - J_*,\quad \frac{\partial \Phi_2}{\partial \tau} + (1-\alpha)\frac{\partial U_2}{\partial x} = J_* - J^*$$
$$\alpha r \frac{\partial U_1}{\partial \tau} + \frac{\alpha r}{\gamma}\frac{\partial P}{\partial x} + f^* = 0,\quad (1-\alpha)\frac{\partial U_2}{\partial \tau} + \frac{(1-\alpha) r}{\gamma}\frac{\partial P}{\partial x} - f^* = 0$$
$$\alpha r C_1 \frac{\partial \theta_1}{\partial \tau} + (1-\alpha) C_2 \frac{\partial \theta_2}{\partial \tau} - \alpha r\left(\frac{1}{\gamma} - \beta\right)\frac{\partial P}{\partial \tau} = (J_* - J^*) L_0$$
$$(1-\alpha) C_2 \frac{\partial \theta_2}{\partial \tau} = (1-\alpha) q^* - LJ^* + \frac{1-r}{\gamma} J_* \tag{2.5}$$
$$P = \theta_1 + \Phi_1/\alpha r + \Phi_2/\alpha,\quad \beta = (C_2 - C_1)\theta_3' + r/\gamma + L'$$
$$f^* = (1-\alpha)\frac{U_1 - U_2}{\tau_v} \quad \left(\tau_v = \frac{\rho_2^\circ d^2 a}{18\mu_1}\right),\qquad q^* = C_2 \frac{\theta_1 - \theta_2}{\tau_T} \quad \left(\tau_T = \frac{\rho_2^\circ d^2 a c_2}{12 k_1}\right)$$
$$J^* = (1-\alpha)\frac{\theta_2 - \theta_3' P}{\tau^\circ} \quad \left(\tau^\circ = \frac{\rho_2^\circ d a}{6 l F^\circ}\right),\qquad J_* = (1-\alpha)\frac{\theta_3' P - \theta_1}{\tau_0} \quad \left(\tau_0 = \frac{\rho_2^\circ d a}{6 l F_0}\right)$$

Here and in what follows, the primes denote the total derivatives with respect to the dimensionless pressure; τ_v, τ_T, τ°, and τ_0 are reduced relaxation times (with the dimensions of a length).

Let us consider the propagation of plane periodic waves in a medium described by the equations (2.5); we shall seek the solutions of this system in the form of a damped traveling wave $\exp[i(kx - \omega t)]$. The condition for the existence of a nontrivial solution of this type leads to the following relationship between the wave vector and the dimensionless perturbation frequency η ($\eta = \omega\tau_T/a$):

$$K^2 = \eta^2 \frac{\alpha\gamma}{\tau_T^2 m_*}\frac{(1 + m - i\eta\tau_v/\tau_T}{(1/\alpha m_* - i\eta\tau_v/\tau_T)}\left(1 + \frac{\Pi_0 + \pi_1\Pi_3 - \Pi_1\Pi_4}{\Pi_2\Pi_4 - \pi_2\Pi_3}\right)$$
$$m = (1-\alpha)/\alpha r,\quad m^* = 1 - \alpha(r-1),\quad \Pi_0 = \operatorname{Re}\Pi_0 + i\operatorname{Im}\Pi_0$$
$$\operatorname{Re}\Pi_0 = (1-r)\left[\theta_3'(C_1 + mC_2) + \beta - \frac{1}{\gamma}\right]\left[(1-\alpha)\left(\frac{1}{\tau_0} + \frac{1}{\tau^\circ}\right) - \frac{(1-\alpha)\tau_T}{C_2\tau_0\tau^\circ}\left(\frac{1-r}{\gamma} - L\right)\right]$$
$$\operatorname{Im}\Pi_0 = \eta(r-1)(1-\alpha)\left[\frac{\gamma\theta_3' C_2 + \gamma L' + r - 1}{\gamma\tau_0} + \frac{\theta_3' C_1}{\tau^\circ}\right] \tag{2.6}$$
$$\Pi_1 = (1-\alpha)\theta_3'\left(\frac{1}{\tau_0} + \frac{1}{\tau^\circ}\right) - i\eta\left(\frac{1}{\gamma} - \beta\right)\frac{\alpha r}{\tau_T},\quad \Pi_2 = (1-\alpha)\frac{L}{\tau_0} - i\eta\frac{\alpha r C_1}{\tau_T},$$
$$\Pi_3 = (1-\alpha)\left(\frac{L}{\tau^\circ} - i\eta\frac{C_2}{\tau_T}\right),\quad \Pi_4 = 1 + \frac{\tau_T}{\tau^\circ}\frac{L}{C_2} - i\eta$$
$$\pi_1 = \frac{\theta_3'\tau_T}{C_2}\left(\frac{L}{\tau^\circ} + \frac{1-r}{\gamma\tau_0}\right),\quad \pi_2 = \frac{1-r}{\gamma C_2}\frac{\tau_T}{\tau_0} - 1$$

Making a passage to the limit from (2.6) we obtain relations for the equilibrium $a^e(\eta \to 0)$ and frozen $a^f(\eta \to \infty)$ velocities of sound in a gas-suspension mixture with phase transitions:

$$a^e = a_1^e G^{1/2} [(G + \theta_3' C_2 m)(1 + m)\alpha^2]^{-1/2}$$
$$\left(G = \frac{L(1-\theta_3')}{1-r} + \theta_3' C_2 + L' + \frac{r-1}{\gamma}, \quad a_1^e = \left[\frac{L}{(1-r)G}\frac{p_0}{\rho_1^\circ}\right]^{1/2}\right) \tag{2.7}$$
$$a^f = a_1^f \left[1 + \frac{1-\alpha}{\alpha}\frac{\rho_1^\circ}{\rho_2^\circ}\right]^{1/2} \quad \left(a_1^f = \left[\frac{C_1}{\beta + C_1/\gamma}\frac{p_0}{\rho_1^\circ}\right]^{1/2}\right)$$

If $\alpha = 1$ (m = 0) we have $a^e = a_1^e$ and $a^f = a_1^f$. It can be seen from (2.7) that $a_1^e \neq a_1^f$, since a^e corresponds to a wave for which the mixture is saturated ($T_1 = T_2 = T_3$); but if $\alpha \approx 1$ and the wave amplitude is nonvanishing, there may be superheated vapor behind the wave ($m = 0$, $\alpha = 1$, $T_1 > T_3$). Allowance for this circumstance eliminates the apparent discrepancy between a_1^e and a_1^f. In a pure vapor, the equilibrium and frozen velocities of sound are the same and equal to a_1^f.

3. A similar treatment applies to the simpler case of a gas-suspension mixture without phase transition when the equations of state of the phases have the form

$$p = \rho_1^\circ R_1 T_1, \quad i_1 = c_{p1} T_1, \quad \rho_2^\circ = \text{const}, \quad e_2 = c_2 T_2 \qquad (c_{p1}, c_2 = \text{const}) \tag{3.1}$$

In this case we have the following relationship between the wave number and the dimensionless perturbation frequency:

$$K^2 = \eta^2 \frac{\alpha}{\tau_T^2 m_*} \frac{(1 + m - i\eta\tau_v/\tau_T)(\gamma C_*/C_1 - i\eta)}{(1/\alpha m_* - i\eta\tau_v/\tau_T)(\Gamma C_*/C_1 - i\eta)}$$
$$\left(\Gamma = \frac{c_{p1} + mc_2}{c_{p1} - R_1 + mc_2}, \quad c_0 = \frac{(c_{p1} - R_1)\alpha\rho_1^\circ + c_2(1-\alpha)\rho_2^\circ}{\alpha\rho_1^\circ + (1-\alpha)\rho_2^\circ}, \quad c_* = c_0(1 + m)\right) \tag{3.2}$$

Here Γ is the effective adiabatic exponent of the mixture and C_0 is the heat capacity of unit mass of the mixture at constant volume.

The expressions for the equilibrium and frozen velocities of sound in such gas suspensions have the form

$$a^e = a\left[\frac{\Gamma}{\gamma\alpha^2(1+m)}\right]^{1/2}, \qquad a^f = a\left[1 + \frac{1-\alpha}{\alpha}\frac{\rho_1^\circ}{\rho_2^\circ}\right]^{1/2} \tag{3.3}$$

The damping coefficient δ in the limit $\eta \to \infty$ tends to the value of δ^f:

$$\delta^f = \frac{1}{2}\left[1 + \frac{1-\alpha}{\alpha}\frac{\rho_1^\circ}{\rho_2^\circ}\right]^{-1/2}\left[\frac{1}{\tau_T^2}\frac{C_*}{C_1}(\gamma - \Gamma) + \frac{1}{\tau_v\tau_T}\frac{m(1-r)^2}{1+mr^2}\right] \tag{3.4}$$

4. It is of interest to estimate the effect of phase transitions on the nature of the dispersion relations.

The curves plotted in Figs. 1 and 2 show how the phase velocity U_p and the damping constant δ depend on the frequency of the external perturbation.

The curves are plotted for different values of the coefficient $F = F^\circ = F_0$ and different vapor volumes in a two-phase steam-water mixture with initial pressure $p_0 = 10$ bar with the following initial thermodynamic data: $a = (\gamma p_0/\rho_{10}{}^0)^{1/2} = 502$ m/sec, $R_1 = 429.5\ \text{J}\cdot\text{kg}^{-1}\cdot\text{deg}^{-1}$; $\mu_1 = 16.05 \cdot 10^{-6}\ \text{N}\cdot\text{sec}\cdot\text{m}^{-2}$, $k_1 = 314.4 \cdot 10^{-4}\ \text{W}\cdot\text{m}^{-1}\cdot\text{deg}^{-1}$, $c_2 = 4.40 \cdot 10^3\ \text{J}\cdot\text{kg}^{-1}\cdot\text{deg}^{-1}$. The curves we have plotted correspond to a particle diameter of $d = 10^{-5}$ m.

The calculations show that if $F \geq 10^{-4}\ \text{kg}\cdot\text{sec}\cdot\text{m}^{-4}$ and $F \leq 10^{-7}\ \text{kg}\cdot\text{sec}\cdot\text{m}^{-4}$ the process is virtually independent of the value of this parameter in the considered range of frequencies. At low frequencies ($\eta \to 0$) and also at fairly high frequencies ($\eta \to \infty$) the effect of F may become more pronounced. Note that the value of F can be determined from a measurement of the propagation velocity and the damping constant of a weak perturbation in a single-component two-phase mixture. However, the absence of reliably tested experimental data prevents us from comparing our results with experimental data.

As the volume of the drops increases, the effect of a departure from phase and temperature equilibrium increases but on the whole remains fairly small. As a rule, the most important process effecting the relations is the friction between the phases. However, as the pressure of the mixture increases, the

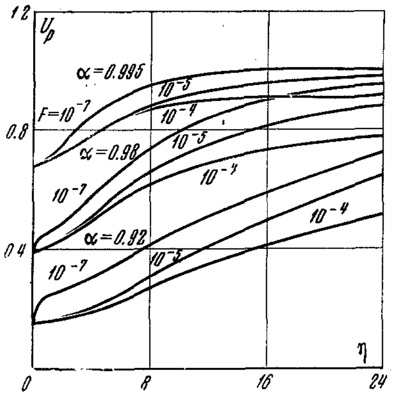

Fig. 1

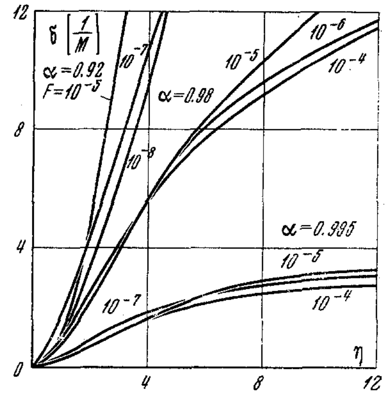

Fig. 2

importance of friction decreases. In the limit $r \to 1$ there is no departure from velocity equilibrium and the dispersion and dissipation due to viscosity vanish.

The relative intensity of the dissipation processes also depends on the relations between the relaxation times of the processes under consideration. Note that the relations between the velocity and temperature relaxation times and the phase-transition relaxation times are determined by the extent to which the medium is dispersive. Under the assumptions we have made concerning the kinetics of vaporization and condensation, a decrease in the dimensions of the dispersed phase is accompanied by an increase in the relative contribution of phase transitions to the energy dissipation and this process may become predominant if the drops have sufficiently small diameters. The effect of mass exchange between the phases on the propagation of a perturbation in a two-phase medium can be analyzed directly by using a single-velocity and single-temperature model of the medium. Note that whereas the phase-transition entails temperature equilibrium the converse is not, in general, true. If the temperatures of the phases are in equilibrium, nonequilibrium phase transitions can nevertheless occur in a wave resulting in dispersion and dissipation. The dispersion relation corresponding to this case has the form

$$K^2 = \eta^2 \frac{\gamma\alpha^2(1+m)}{\tau_T^2}\left[\frac{2m(1-r)(G + C_2\theta_3' m)\tau_T/\tau_0 - i\eta(Q + C_2 m)}{2mL\tau_T/\tau_0 - i\eta(C_1 + mC_2)}\right] \tag{4.1}$$
$$(Q = G + (1-\theta_3')(C_1 - L/(1-r)))$$

Here G and the equilibrium velocity of propagation of a perturbation are calculated in accordance with (2.7). For the velocity of sound frozen with respect to mass exchange we have the expression

$$(a_*^f)^2 = \frac{a^2}{\gamma\alpha^2(1+m)}\frac{C_1 + mC_2}{C_1(1-\theta_3') + C_2(m+\theta_3') + L' + (r-1)/\gamma} \tag{4.2}$$

It can be seen from (2.7) and (4.2) that $a^f \neq a_*^f$, since a_*^f is calculated from a mixture that has velocity and temperature equilibrium. In a pure vapor ($\alpha = 1$) we have $a_1^f = a_{1*}^f$.

Some estimates show that the coefficients F in the linear relations for the rates of the phase transitions are very large. Therefore, at sufficiently low frequencies of the external perturbation a single-component two-phase medium has an equilibrium phase composition and satisfies the condition of equality of the phase temperatures. In this case, dispersion and dissipation are due exclusively to the viscous interaction of the phases and the dispersion relation is

$$K^2 = \eta^2 \frac{1}{(U_1^e)^2}\frac{\alpha}{\tau_T^2 m_*}\frac{(1 + m - i\eta\tau_v/\tau_T)}{(1/\alpha m_* - i\eta\tau_v/\tau_T)} \tag{4.3}$$

Note that for $r < 0.1$ allowance for the added mass does not appreciably affect the dispersion relations. Taking into account this observation, we write the dispersion relation (3.2) in the form

$$K^2 = \eta^2 \frac{\alpha}{\tau_T^2 m_*}\frac{[1 + m - i\eta(1 + 0.5r(1+m))\tau_v/\tau_T][\gamma C_*/C_1 - i\eta]}{[1/\alpha m_* - i\eta(1+0.5r)\tau_v/\tau_T][\Gamma C_*/C_1 - i\eta]} \tag{4.4}$$

Any small perturbation of arbitrary profile can be obtained by superimposing harmonic waves. In accordance with the general results given, for example, in [8] for the case of a reacting gas, we can assert that the leading edge of an arbitrary pulse moves with velocity a^f and is damped exponentially with the exponent $(-\delta^f x)$.

Finally, note that the expressions (2.7) and (3.3) for the equilibrium and frozen velocities of sound agree with the relations that follow from a consideration of the conditions of existence of densification waves in two-phase media [9].

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